On the Threeven and the Throdd
Threeven, Throdd , Triangland , Trivision

Trecedent

The author of the polysigned numbers, ゴールデン・P・ティム, had given possible hints to the question 'what is beyond two signs'. On this line of research flutters this document.

On the Threeven and the Throdd

The name of this section got his reason after a Reddit's thread titled 'Threeven and Throdd: Words to Describe Numbers Divisible/Not Divisible by Three?'[1].

The redditor TickTak wrote

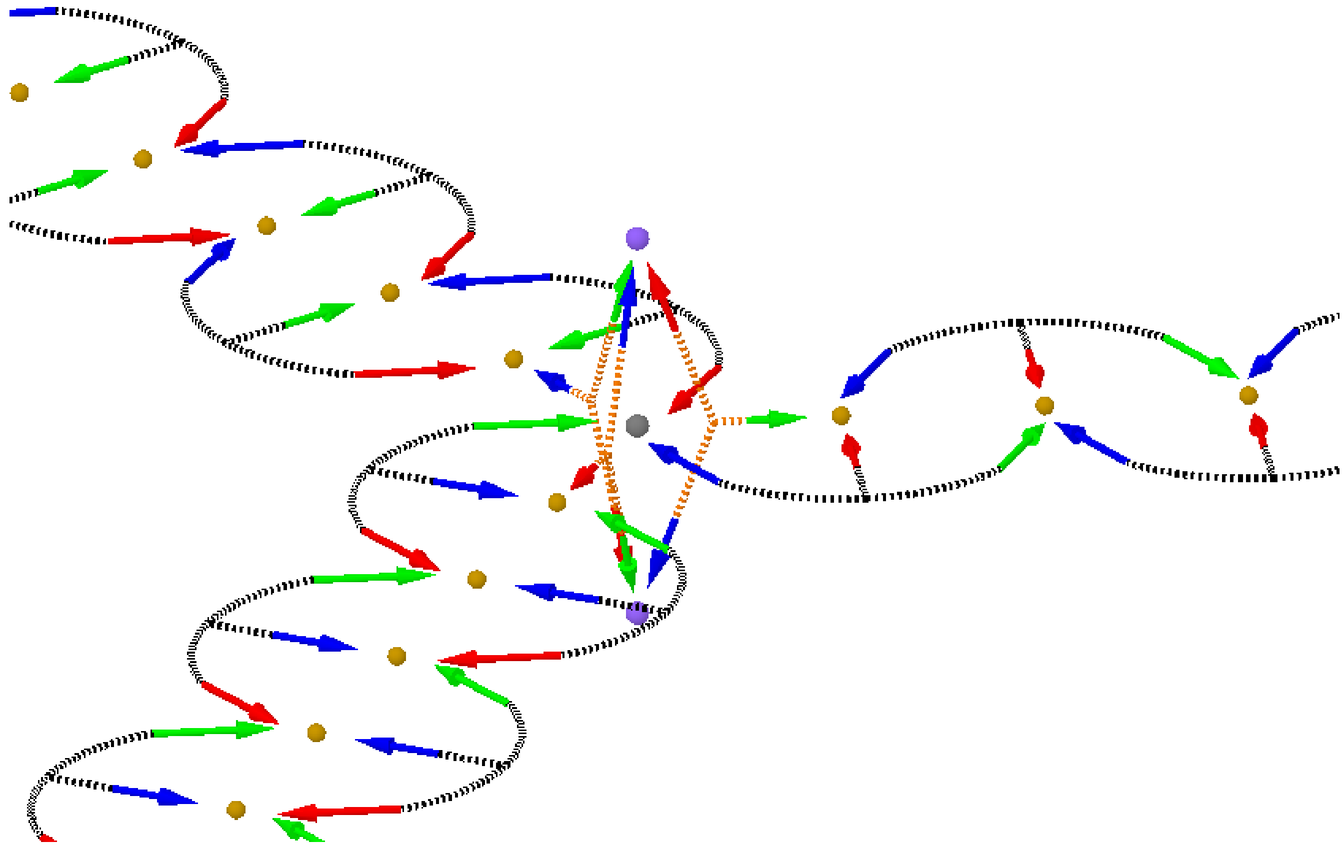
> So threeven is mod3-0, throdd is mod3-1,2. Now we will need two new words.
> thronly - mod3-1 - three + only/lonely (from “one is the loneliest number”)
> thrompany - mod3-2 -three + company (from “two’s company, three’s a crowd”)
> Extending:
> fourven - mod4-0 / fourdd - mod4-1,2,3
> fournly- mod4-1
> fourpany - mod4-2
> fourwded - mod4-3 - four + crowded

and the redditor ITBlueMagma wrote

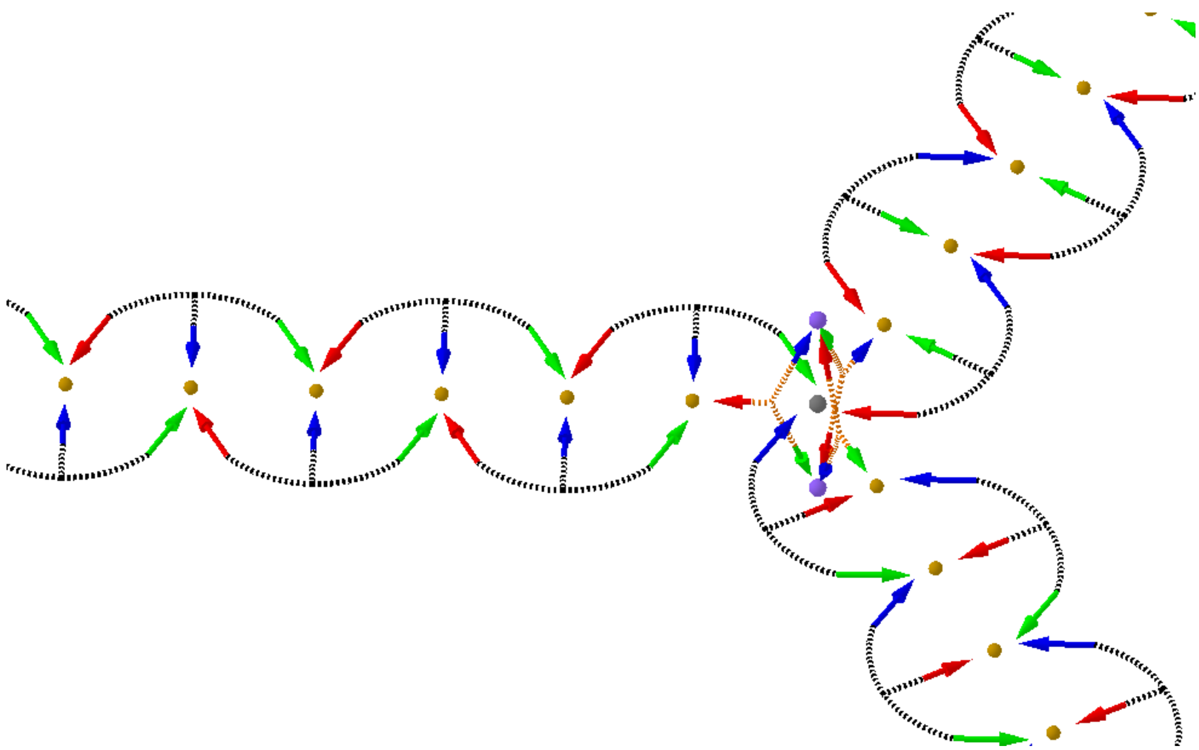
> So after thinking about it for some time, I feel using number as prefix sound weird, I would probably prefer using greek prefixes: tri, tetra, penta, hexa and so on.
> Then I feel, odd means, one left over, the "odd one out"
> For thah reason I would have:
> 0mod2: even
> 1mod2: odd
> 0mod3: triven (tri + even), I pronouce it like try-ve-n
> 1mod3: triodd (tri + odd) I pronounce try-odd
> 2mod3: tricond (tri + second)
> 0mod4: tetraven (tetra + even)
> 1mod4 tetraodd (tetra + odd)
> 2mod4: tetracond (tetra + second)
> 3mod4: tetrahird (tetra + third), I pronounce tetra-heard

At first sight, reminiscences of the Eisenstein Integers (see [2]and[3]) may comes to mind, in case the reader is more acquainted with number theory. It can be also the case if hexagonal grid for programmers, like [4].

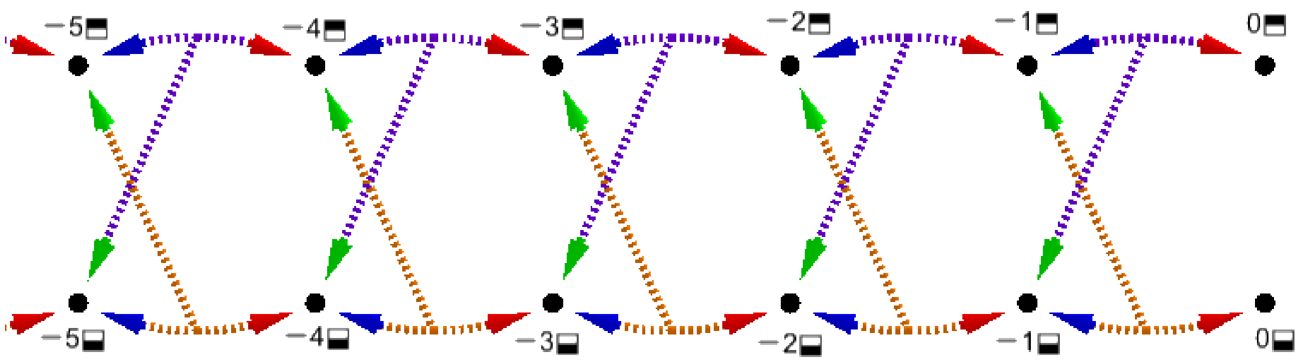
Alluding to the mathematical metaphor that the above post captures, a couple of versions of a Threeling-wise Number Line (of integers) is presented, based on the partial sketches of the Ternary Frog. It is up to the reader to figure out how the dynamics (or trynamics) of a threeling succesor function will be for the tritonic integers.



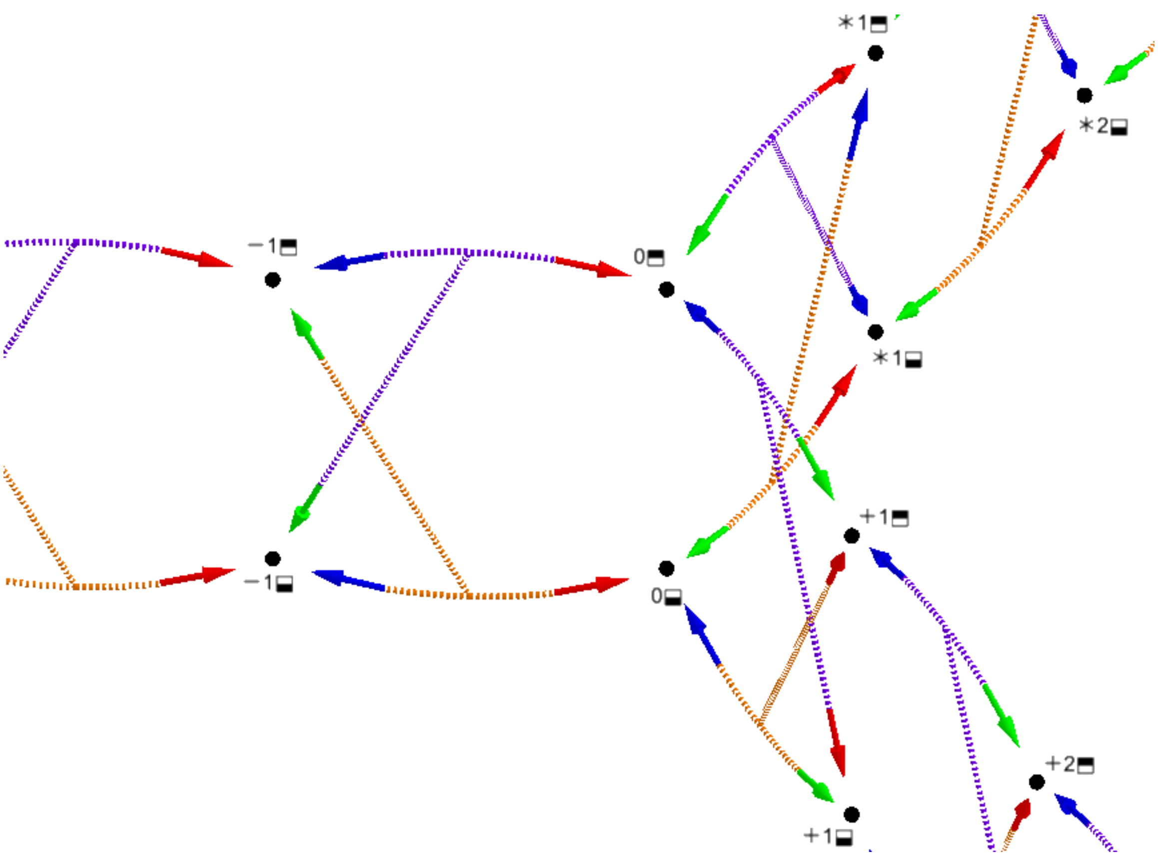
Perhaps, good way to put it, will be to imagine a Tritraversable Traffic System, that is, imagining a tritonic version of the bidirectional traffic system (the streets where cars, buses and trucks go by), but this time, an aereal pathway for planes, helicopters and drones, a sort of discrete aereal three-way system.



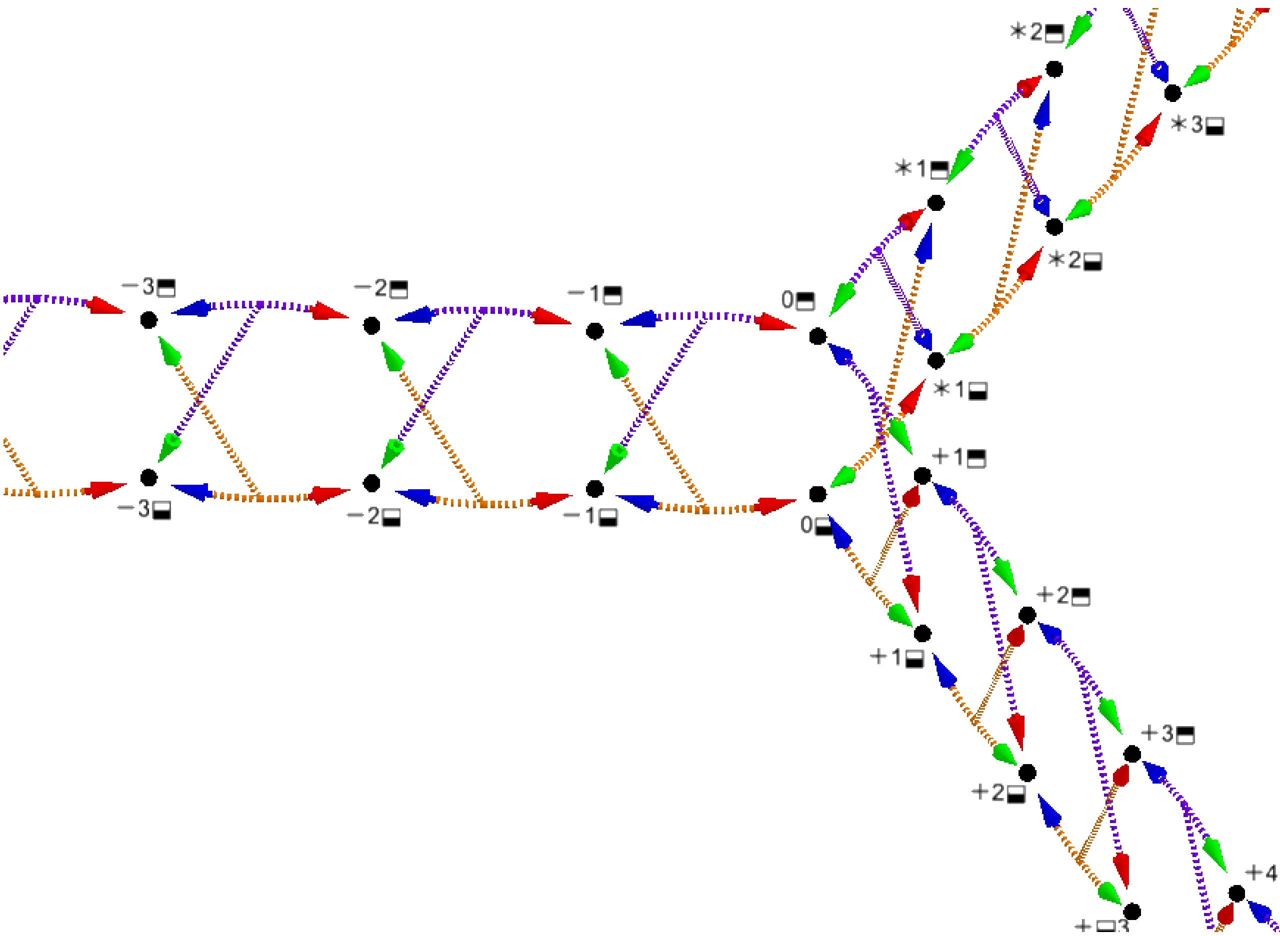
And Second type of integer number line



A zoom of the center



A panoramic view



Not actually DNA Helices, rather a three-signed (or triply-threaded) number line.

It is unknown to the author if ordinality can be leveraged to access cardinality.

Triangland, Squareland and The Figural Area

The word 'triangland' comes from a little story written by Jacob M. Cohen called ‘Triangland: Where Numbers are Triangled Instead of Squared’

The following is an extract

Welcome to Triangland
Triangland (technically, the Triunited Kingdom) is a place much like our own, but with one major difference: instead of squaring numbers, people triangle numbers. If you ask the average Trianglish elementary school student what "three squared" is, they won't know the answer, but ask them for "three triangled" and they'll instantly respond "six." This blog post will cover some of the cool features of triangling numbers that every schoolchild in Triangland learns, culminating in the Trianglish Quadratic Formula.

At https://beautifulthorns.wixsite.com/home/post/triangland-where-numbers-are-triangled-instead-of-squared

It is the usual that the number of measurements to evaluate an extension (of a planar region) is two, the length and width. Commonly associated with the rectangle and the square, and more specifically, linked to square numbers.

But, what about a concept of area inextricably linked to the triangle instead of the square, that is, where the triangular numbers are the protagonist ?
If such is the case, what is known as area would be just a special case among other cases of 'Figural Area'. What makes squares and rectangles more suitable ?

We search something that manage to capture invariance.

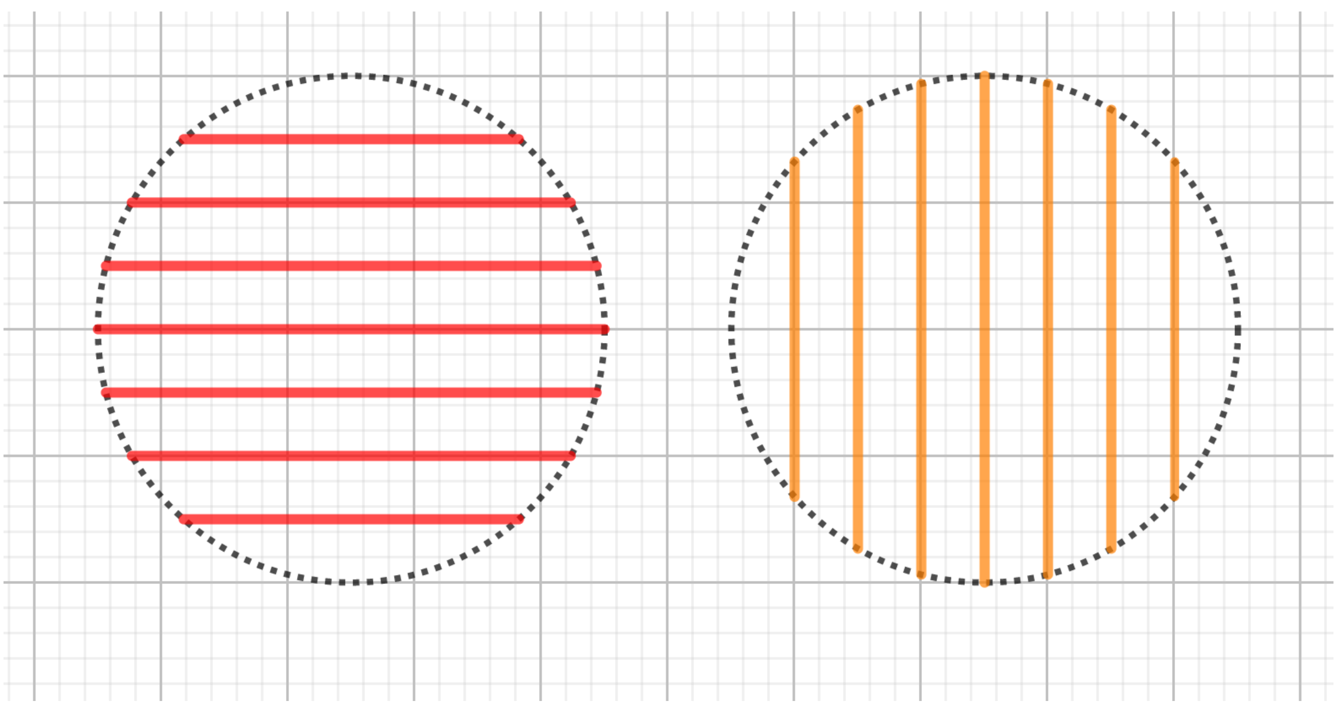
Certainly, several authors, like Wayne Roberts, Kirby Urner and Luis Teia speak beyond units of area based on just squares and cubes, that, is where the triangle and other types of units (in the case of space) are considered.

But, one piece is missing, and it is not exactly the topic of Dissection of Regions of the plane (or the space) into smaller ones.

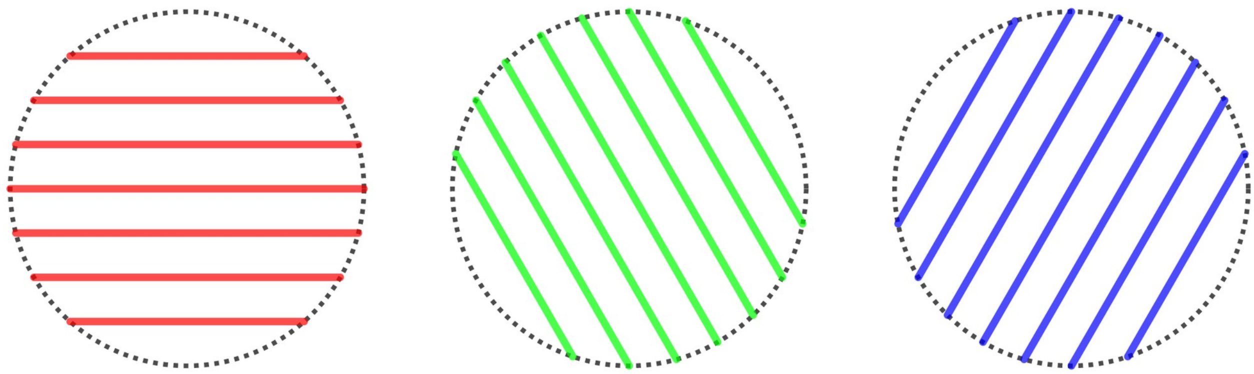
It's usual the approximation of an irregular area by rectangles, either vertical or horizontal. As well as with other figures like triangles, hexagons, and others.

On the Triparallelian Area

Not much time ago, a geometer (John Gabriel) uploaded a video called 'General definition of area and volume' where he explains the notion of area as a product of arithmetic means. For the plane, will be the case that, the arithmetic mean of horizontal line segments, multiplied by the arithmetic mean of vertical line segments (of a planar region) yields the area of the region in question. This is not about dissections into rectangles, but about line segments.

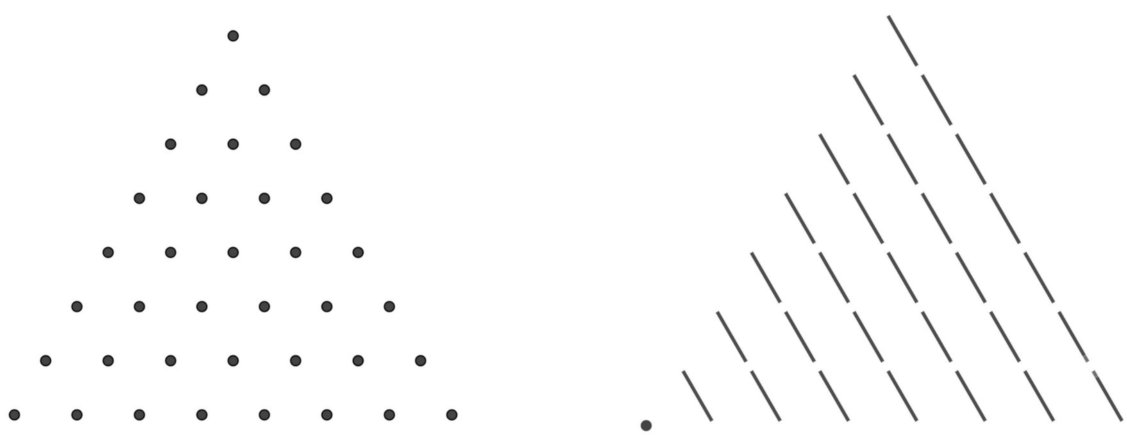


Now, with a brief modification, the above can be transformed. Instead of two sets of parallel line segments (relative each other by 90°), we can use three sets of parallel line segments (relative each other by 120°/60° this time). With this, we define the triparallelian area.



The Triparallelian Area is a type of Figural Area where the natural arity of the product is of three. Third powering in the plane, for multiples of an equilateral triangle you may say. A product of three arithmetic means yield the triparallelian area (of any region in the plane).

The triparallelian area also has a preferred object, which is, The Triangle. Simply look the image.



Too much hints and clues, enough for now. Just imagine triangular number as showed in the right, not as showed in the left. You are ready to deduce and compute the triparallelian area of a triangle. This time, the (triparallelian) area of a equilateral triangle is not longer irrational !!!

A case against the universality of the Circle and the Square : A Double Conspiracy

The blood of the circle and the hyperbola is orthogonal (perhaps you could call

it of 'even heritage'), that the exponential e^x and the Euler formula are suspicious objects in Triangland, so new contants are needed beyond pi and e.

The test is simple, compute the triparallelian area of the circle, hyperbola, triangle, square, pentagon, hexagon. who is exuding unexpected irrationality ? and who is exuding a newfound rationality now?

For the triparallelian area, the parahexagon whose inner angles are 120° replaces the rectangle. You may study this parahexagons with integer sides first. Additionaly, the topic of parallelotopes seems promising in this context.

Notice that the triparallelian area seems a notable candidate for the integration of functions in quadrays/polysigns.

A perfect blend of cartesian and fermatian flavors.

As a sidenote, a good text on Figurate numbers is the book 'Figurate Numbers' written by Elena Deza and Michel Marie Deza.

On the Terfractions and Tridentity

The tridentity relation and terfractions are presented in 'intrifix' format, since a natural ternary notation is not easy to adopt in the present text.

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                                                              a
For example, a possible notation for terfractions may be    ---------
                                                             b | c
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but a practical notation is on the form a ⫽ b ⫻ c , but, do not be deceived, it is just a convenient optical trick in order to depict 'trivision' (term after the ternary frog).

The Addition of Terfractions

∗(a ⫽ b ⫻ c) @ ∗(d ⫽ e ⫻ f) ≡ ∗g ≡ ∗h

∗(a ⫽ b ⫻ c) ≡ ∗g @ −(d ⫽ e ⫻ f) ≡ ∗h

∗(a ⫽ 1 ⫻ c) ≡ b·(∗g @ −(d ⫽ e ⫻ f)) ≡ ∗h

∗(a ⫽ 1 ⫻ c) ≡ ∗bg @ −(bd_⫽ e ⫻ f) ≡ ∗h

∗(a ⫽ 1 ⫻ 1) ≡ ∗bg @ −(bd ⫽ e ⫻ f) ≡ ∗ch

∗(a ⫽ 1 ⫻ 1) = ∗a

∗a ≡ ∗bg @ −(bd ⫽ e ⫻ f) ≡ ∗ch

∗a @ ∗(bd ⫽ e ⫻ f) ≡ ∗bg ≡ ∗ch

∗(bd ⫽ e ⫻ f) ≡ ∗bg ≡ ∗ch @ +a

∗(bd ⫽ 1 ⫻ f) ≡ ∗ebg ≡ ∗ch @ +a

∗(bd ⫽ 1 ⫻ 1) ≡ ∗ebg ≡ f·(∗ch @ +a)

∗(bd ⫽ 1 ⫻ 1) ≡ ∗ebg ≡ ∗fch @ +fa

∗(bd ⫽ 1 ⫻ 1) = ∗bd

∗bd ⩦ ∗ebg ⩦ ∗fch @ +fa

∗bd @ ∗fa ⩦ ∗ebg ⩦ ∗fch

(∗bd @ ∗fa) ⫽ eb ⫻ fc ⩦ ∗g ⩦ ∗h

And with the first and the last sentences

∗(a ⫽ b ⫻ c) @ ∗(d ⫽ e ⫻ f) ⩦ ∗g ⩦ ∗h

(∗bd @ ∗fa) ⫽ eb ⫻ fc ⩦ ∗g ⩦ ∗h

One is very tempted to write

∗(a ⫽ b ⫻ c) @ ∗(d ⫽ e ⫻ f) = (∗bd @ ∗fa) ⫽ eb ⫻ fc

Very relevant to the topic are the following references

I invented a NEW number system! written by the user RLVideosgunner
at https://www.youtube.com/watch?v=ZvI4W0dDvlk

Trirational numbers in the complex plane (and generalizations) written by the user
Francis Ocoma at https://math.stackexchange.com/questions/4520435/trirational-numbers-in-the-complex-plane-and-generalizations,
https://focoma.blogspot.com/2022/10/abstract-algebra-adventures-part-2.html
and https://focoma.blogspot.com/2022/10/abstract-algebra-adventures-part-3.html

Unary Operations on Homogeneous Coordinates in the Plane of a Triangle written by
Peter J. C. Moses and Clark Kimberling (see Binary Operations on Inverse Pairs)
https://www.researchgate.net/publication/382137157_Unary_Operations_on_Homogeneous_Coordinates_in_the_Plane_of_a_Triangle

One could rewrite the above reasoning using other than T. Golden notation.
This time the frog style will be used.

@ <--> + (usual symbol for addition)
∗1 <--> +1 (positive one of the real number line)
−1 <--> Δ1 (first cubic root of one, usually a symbol w or ω is used)
+1 <--> ϟ1 (second cubic root of one, usually a symbol w² or ω² is used)

(a ⫽ b ⫻ c) + (d ⫽ e ⫻ f) ⩦ g ⩦ h

(a ⫽ b ⫻ c) ⩦ g + Δ(d ⫽ e ⫻ f) ⩦ h

(a ⫽ 1 ⫻ c) ⩦ b·(g + Δ(d ⫽ e ⫻ f)) ⩦ h

(a ⫽ 1 ⫻ c) ⩦ bg + Δ(bd ⫽ e ⫻ f) ⩦ h

(a ⫽ 1 ⫻ 1) ⩦ bg + Δ(bd ⫽ e ⫻ f) ⩦ ch

(a ⫽ 1 ⫻ 1) = a

a ⩦ bg + Δ(bd ⫽ e ⫻ f) ⩦ ch

a + (bd ⫽ e ⫻ f) ⩧ bg ⩧ ch

(bd ⫽ e ⫻ f) ⩧ bg ⩧ ch + ϟa

(bd ⫽ 1 ⫻ f) ⩧ ebg ⩧ ch + ϟa

(bd ⫽ 1 ⫻ 1) ⩧ ebg ⩧ f·(ch + ϟa)

(bd ⫽ 1 ⫻ 1) ⩧ ebg ⩧ fch + ϟfa

(bd ⫽ 1 ⫻ 1) = bd

bd ⩧ ∗ebg ⩧ ∗fch + ϟfa

bd + fa ⩧ ebg ⩧ fch

(bd + fa) ⫽ eb ⫻ fc ⩧ g ⩧ h

Again, with the first and the last sentences

(a ⫽ b ⫻ c) + (d ⫽ e ⫻ f) ⩧ g ⩧ h

(bd + fa) ⫽ eb ⫻ fc ⩧ g ⩧ h

we may simply utter (now in the classical format)

(a ⫽ b ⫻ c) + (d ⫽ e ⫻ f) = (bd + fa) ⫽ eb ⫻ fc

Compared to its binary analog

(a/b) + (c/d) = e
(a/b) = e + -(c/d)
(a/1) = be + -(bc/d)
where (a/1) = a
a = be + -(bc/d)
a + (bc/d) = be
(bc/d) = be + -a
(bc/1) = dbe + -da
where (bc/1) = bc
bc = dbe + -ad
bc + ad = dbe
(bc + ad)/db = e

A couple of small examples

12 ⩧ 12 ⩧ 12
12 ⩧ 6·2 ⩧ 4·3
12 ⫽ 2 ⫻ 3 ⩧ 6 ⩧ 4

7 ⩧ 7 ⩧ 7
7 ⩧ 3 + 4 ⩧ 5 + 2
7 + ϟ4 + Δ5 ⩧ 3 ⩧ 2

If the reader plays a bit with the above tridentities, very soon will find that the tridentity possess a couple of strange features. The compatibility between a three-signed arithmetic and the tridentity, is at first sight similar to the compatibility of a two-signed arithmetic with the identity.

It is possible to effectively move an artifact from one side of the tridentity

to other side (where the sign of the artifact is modified according the side), and it's possible to not necessarily resort to 'apply the same operation on every side of the equation', this time, different from 'moving and object from one side to another', as the total of sides of equality/equation in question is now three, not just two. Although this produces local disequilibria among sides, the global equilibrium is maintained (as asserted by the ternary frog).

The ternary frog does not clarify if $a \mathbin{/\!/} b \mathbin{/\!/\!/} c$ is similar to $(a^{+1})(b^{w})(c^{w^2})$ where w is the cube root of unity in C, since the analogy of cubic roots of unity in $\mathbb{C}$ is a well-known arithmetic, that is, the usual p3 sign table, but, it need not be. Just keep in mind that you must define the geometry/topology where the tridentity will work.

The ternary frog goes on to list claims related to some sort of empowerment of the Analytic Geometry (of spatial lines and curves) by means of the trivision, namely, the use of trivision to capture the essence of three-dimensional slopes. In any case, besides of considering it a kind of tribute to Al-Jabr of Al-Khwarizmi, it can be valued as a curiosity, presumably, to research some sort of generalization of Egyptian Fractions, Dyadic Rationals, to create puzzles of Recreational Mathematics with an extra evil twist of difficulty, or to do a continuation of Baldor's Algebra.

Finding a triangular or ternary analogs of a mathematical objects does seem a good exercise to endure harsh winters.

Resources

[1] Threeven and Throdd: Words to Describe Numbers Divisible/Not Divisible by Three?https://www.reddit.com/r/dozenalsystem/comments/huf4a9/threeven_and_throdd_words_to_describe_numbers/
[2] Rasmus Frigaard Lemvig, A tour of the Eisenstein integers , https://rasmusfl.github.io/Documents/EI.pdf
[3] Emily Gullerud, Aba Mbirika (2019 – 2020), An Euler phi function for the Eisenstein integers and some applications , https://arxiv.org/abs/1902.03483
[4] Amit Patel, Explanation of Hexagonal Grids (1995–2024), http://www-cs-students.stanford.edu/~amitp/gameprog.html
[5] Ternary arithmetic, factorization, and the class number one problem - Aram Bingham

バッタ たなか signing off 。
July 26 + @ + 半


```
// 'herd' instead of 'crowded' ?

/*
* Other possible alternative for the longitude of a line segment of length A
* will be to associate (to it) other 'figural length' of length A² (the 'parallels'
* and the 'antiparallels'
*/
```